A FAC potential for AUTOSTRUCTURE.

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1 Introduction

This document describes features available in AUTOSTRUCTURE (AS) that require use of the current development version (30.x) which can be found at: http://amdpp.phys.strath.ac.uk/autos/autos-30/ and requires the usual userid and password to access it — ask me if you have forgotten them. The discussion below assumes a working knowledge of AS and familiarity with the contents of the current WRITEUP file.

1.1 Motivation

The Flexible Atomic Code (FAC) of Gu (2008) is in widespread use. As such, one can encounter the questions:

'Why are you not using FAC?'

and

'Why do your results differ from those of FAC?'

This working note attempts to help you answer these questions.

(Note: we are concerned here only with the calculation of atomic data and not plasma population modelling.)

1.2 Background

AS and FAC have a key common feature — they are both 'distorted-wave' codes, i.e., they solve uncoupled radial equations.

This distinguishes them from Hartree–Fock and Dirac–Fock structure codes, which solve coupled radial equations. And similarly, collision codes such as R-matrix.

FAC is fully relativistic: it solves the two-component Dirac radial equation. It includes the Breit interaction in the Hamiltonian (+QED).

AS is semi-relativistic: it solves the kappa-averaged Dirac equation (and usually neglects the small component, renormalizing the large component instead). It includes the Breit interaction in the Pauli approximation in the Hamiltonian (+QED).

The K-shell of Uranium aside, these differences are not important, in general.

As always, the configuration interaction (CI) expansion is key — the larger the CI expansion, the more complete our basis.

But, if we use the same CI expansion, what is the main source of difference between AS and FAC then?

The answer lies in the distorted-wave potential(s) used by the two codes.

The key difference between FAC and AS is that FAC uses a unique potential.

The use of unique potential gives rise to a unique set of orthogonal orbitals. But, no matter whether they be bound, continuum, or are used to represent both N- and (N-1)-electron atoms and ions in describing bound-free atomic processes (e.g. autoionization, photoionization), all orbitals are described by the same single distorting potential within FAC.

In contrast, while AS can use a unique potential such as Thomas–Fermi, it is free to use different potentials as it or the user sees fit: for bound vs continuum or different charge states. The use of such potentials gives rise both to non-orthogonal and/or non-unique orbitals. This (apparently) gives rise

to a much more complicated atomic structure problem which needs to take account of all the overlap integrals, and which disappears on using a unique potential.

However, we follow the approach of Cowan (1981) whose Hartree-Fock Relativistic (HFR) approach is inherently non-orthogonal (which follows directly from solving the kappa-averaged HFR equations). This approach involves taking all bound orbital overlap integrals to be zero or unity based-on whether $n \neq n'$ or n = n', for l = l'. AS includes the resulting exchange overlap integrals for the continuum as well. It is justified by the long experience with HFR (and AS) in comparison of their results with those from HF and DF codes, as well as experiment.

The flexibility of AS with regards to potentials and the orbitals they describe partways answers the questions raised at the start. However, a more powerful response is to be able to reproduce the unique FAC potential within AS and hence obtain comparable results. One is then free to use such or to argue for a different choice of atomic structure available within AS, but not FAC.

However, constructing the FAC potential within AS requires a little work and understanding.

We first note FAC versions in use. The last official release of FAC is v1.1.4 dated 2016.11.9. Most users make use of the unreleased v1.1.5. Specifically, for this work we used FAC v1.1.5 dated 2023.03.23 from GitHub, while an earlier version of 1.1.5 used is dated 2020.04.20 on Github. We refer to these versions unofficially as FAC 2020 and FAC 2023.

2 Constructing a FAC Potential for AS — part 1 (a history lesson)

Like the Slater-Type-Orbital (STO) potential (Burgess et~al, 1989) available in AS for some time, the FAC potential is based on a shell-structure description of the ion in question. It is actually closely related to the average-energy of a configuration.

We start-off by detailing the potential V_{α} seen by an electron (of orbital) α , in a configuration of an *N*-electron ion due to the other N-1 electrons in the same configuration. Let ω_{α} denote the occupation numbers which define the configuration. Then $\sum_{\alpha} \omega_{\alpha} = N$. In the configuration-average approximation (Cowan, 1981)

$$V_{\alpha}(r) = \sum_{\beta} (\omega_{\beta} - \delta_{\alpha\beta}) Y^{0}_{\beta\beta}(r) - \sum_{\beta} (\omega_{\beta} - 1) \delta_{\alpha\beta} \sum_{\lambda > 0} f^{\lambda}_{\beta\beta} Y^{\lambda}_{\beta\beta}(r) - \sum_{\beta} (1 - \delta_{\alpha\beta}) \omega_{\beta} \sum_{\lambda} g^{\lambda}_{\alpha\beta} Y^{\lambda}_{\alpha\beta}(r) \rho_{\alpha\beta}(r) / \rho_{\alpha\alpha}(r)$$
(1)

where the sum over β is over all orbitals (including α) of the configuration in question.

The corresponding interaction energy (E_{α}) between an electron α and all other electrons is given by

$$E_{\alpha} = \int_{0}^{\infty} V_{\alpha}(r) \rho_{\alpha\alpha}(r) \mathrm{d}r \,. \tag{2}$$

Then,

$$\bar{E} = \frac{1}{2} \sum_{\alpha} \omega_{\alpha} E_{\alpha} \tag{3}$$

is the total electron-electron Coulomb average-energy of the configuration. Here,

$$Y^{\lambda}_{\alpha\beta}(r) = \int \frac{r^{\lambda}_{<}}{r^{\lambda+1}_{>}} \rho_{\alpha\beta}(r') \mathrm{d}r' \tag{4}$$

where $r_{<}, r_{>} = \min, \max(r, r')$, respectively, and $\rho_{\alpha\beta}$ is the probability density, given by

$$\rho_{\alpha\beta}(r) = P_{\alpha}(r)P_{\beta}(r) + \{Q_{\alpha}(r)Q_{\beta}(r)\}$$
(5)

(and where the small component Q_{α} may, or may not, be included.) Finally, if α represents $n_{\alpha}l_{\alpha}$ (e.g. in AS) then

$$f_{\alpha\alpha}^{\lambda} = \frac{2l_{\alpha} + 1}{4l_{\alpha} + 1} \left(\begin{array}{cc} l_{\alpha} & \lambda & l_{\alpha} \\ 0 & 0 & 0 \end{array}\right)^2 \tag{6}$$

and

$$g_{\alpha\beta}^{\lambda} = \frac{1}{2} \left(\begin{array}{cc} l_{\alpha} & \lambda & l_{\beta} \\ 0 & 0 & 0 \end{array} \right)^2 \tag{7}$$

while if α represents $n_{\alpha}l_{\alpha}j_{\alpha}$ (e.g. in FAC) then

$$f_{\alpha\alpha}^{\lambda} = \frac{1}{2j_{\alpha}+1} \left(\begin{array}{cc} j_{\alpha} & \lambda & j_{\alpha} \\ -\frac{1}{2} & 0 & \frac{1}{2} \end{array}\right)^2 \tag{8}$$

and

$$g_{\alpha\beta}^{\lambda} = \begin{pmatrix} j_{\alpha} & \lambda & j_{\beta} \\ -\frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}^{2}.$$
 (9)

We note that line 1 in equation (1) is just the static potential term used by the STO potential of Burgess *et al* (1989) which is incorporated into AS. It can be evaluated analytically when the $P_{\alpha}(r)$ are STOs. The introduction of adjustable scaling parameters (λ_{α}) in this analytic potential combined with the minimization of $\langle H \rangle$ means the remaining two lines (due to equivalent electrons and exchange, respectively) can be neglected.¹

Instead, let us consider using the STO potential (flagged by NZION< 0 in NAMELIST SMINIM) to start-off a self-consistent iteration (PPOT='SCCA' in SMINIM) using the full expression in (1). The occupation numbers ω_{α} are defined as in the STO case. For example, for each orbital α the user flags (via MCFMX in SMINIM) the configuration to be used by AS to define the occupation numbers to be used in (1) for that orbital. In the case of fully-relaxed orbitals (BASIS='RLX' in SALGEB) the occupation numbers are trivially known by AS. In all instances described so far, the potential is orbital dependent due to (1) taking account of self-screening, i.e., if $\sum_{\alpha} \omega_{\alpha} = N$ then $rV_{\alpha}(r) \to N-1$ as $r \to \infty$.

A unique potential $V_{\alpha} = V$ can be imposed by the user defining (NOCC via SMINIM) occupation numbers such that $\sum_{\alpha} \omega_{\alpha} = N - 1$ and the account of self-interaction in (1) is then switched-off. The usual choice for the occupation numbers ω_{α} in this instance is those of the ground configuration of the next ionization stage — see the discussion in Tanaka *et al* (2020) in relation to HULLAC (Bar-Shalom *et al* 2001) and AS Working Notes No. 1 (Badnell 2022) in relation to AS.

FAC takes an alternative approach (Gu, 2008). It forms a unique potential by averaging the V_{α} of (1) over the probability densities $\rho_{\alpha\alpha}$ (and occupation numbers ω_{α}) viz.

$$V(r) = \frac{\sum_{\alpha} \omega_{\alpha} V_{\alpha}(r) \rho_{\alpha\alpha}(r)}{\sum_{\alpha} \omega_{\alpha} \rho_{\alpha\alpha}(r)}$$
(10)

and

$$\frac{1}{2} \int_0^\infty V(r) \sum_\alpha \omega_\alpha \rho_{\alpha\alpha}(r) \mathrm{d}r = \frac{1}{2} \sum_\alpha \omega_\alpha E_\alpha = \bar{E}$$
(11)

is again the electron-electron Coulomb average-energy of the configuration defined by ω_{α} .

¹The addition of a local exchange STO potential to the static one is an option in AS.

If the user supplies a unique set of occupation numbers ω_{α} such that $\sum_{\alpha} \omega_{\alpha} = N$ then again $rV(r) \rightarrow N-1$ as $r \rightarrow \infty$, by inspection of (1) and (10).

The choice as to whether N corresponds to the recombined or recombining ion in bound-free problems is not set by FAC, the user must choose — Gu (2008) recommends the use of N rather than N - 1. In contrast, if the user does not specify the occupation numbers then FAC uses the mean of the subconfiguration occupation numbers meant over all subconfigurations.

This completes the detailing of the original unique FAC potential described by Gu (2008) now available in v30.x of AS. It can be switched-on by adding FAC='OLD'² to SMINIM, together with NOCC occupation numbers³ such that $\sum_{\alpha} \omega_{\alpha} = N$, so as to obtain screening by N - 1 electrons.

Although NZION< 0 and PPOT='SCCA' are implicitly required, the flagging of FAC operation by the user is sufficient. AS automatically switches them on necessarily so. In addition, AS also automatically sets all flags necessary to align its operation with those of the FAC default operation. Note, the default user cannot accidentally re-set such to conflict with FAC. An expert flag is available for developers to override this operation.

3 Constructing a FAC Potential for AS — part 2 (the present day)

Despite Gu (2008) the FAC potential just described has not been used by FAC since its early days (Gu, 2023). Gu (2003) already noted the use by FAC of a Dirac-Hartree-Slater (DHS) potential. The basic DHS potential (or Dirac-Fock-Slater, DFS) is the same as that described in detail by Cowan (1981, Sec 7.11) under the Hartree-Fock-Slater (HFS) method. The analogous Dirac form is described by Sampson *et al* (2009).

Using the notation introduced in part 1, the unique DHS potential is given in terms of the direct static term again plus a *local* exchange potential

$$V(r) = \sum_{\beta} \omega_{\beta} Y^{0}_{\beta\beta}(r) - a \frac{3}{2} \left[\frac{24}{\pi} \rho(r) \right]^{1/3}.$$
 (12)

The term in $\rho(r)$ without subscripts is the total spherically averaged electron number density, viz.

$$\rho(r) = \frac{1}{4\pi r^2} \sum_{\alpha} \omega_{\alpha} \rho_{\alpha\alpha} , \qquad (13)$$

such that (dV is the volume element here)

$$\int \rho(r) \mathrm{d}V = N \,. \tag{14}$$

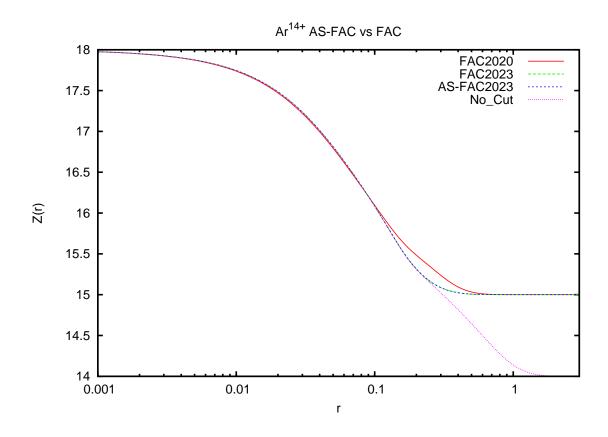
The coefficient a = 1/2 a.u. in equation (12) gives the original Slater derivation. Subsequently, the Kohn-Sham (1965) value of a = 1/3 a.u. was found to give better agreement with Hartree-Fock — see Cowan (1981) for a detailed discussion. Sampson *et al* (2009) adopt a = 1/3 a.u. as well.

More generally, a can be treated as an optimizable parameter. Indeed, FAC uses a look-up table of values for a which were pre-optimized on the ground configuration of the ion defined by Z and N. Actually, FAC collects all the numerical factors together in terms of a variable AHX a.u. (i.e. $AHX = (1/3)(3/2)(6/\pi^2)^{1/3} = 0.4235$ a.u. for the above) and it is this rather than a which is held in the look-up table.

The FAC look-up table has been ported into AS.

²FAC='NO' is the default for PPOT='SCCA', which then uses the V_{α} potentials of equation (1).

³Only the occupation numbers for the valence orbitals should be supplied.



It is also possible for the user to set their own value of AHX directly, using the AS variable FAC_AHX in SMINIM. 4

As is well known, the drawback with 'Hartree-Slater' methods is that for $\sum_{\alpha} \omega_{\alpha} = N$

$$rV(r) \to N \qquad \text{as} \qquad r \to \infty,$$
 (15)

instead of the desired value of N-1.

Rather than the user supplying occupation numbers $\sum_{\alpha} \omega_{\alpha} = N - 1$, the FAC default is to make use of the Latter (1955) cut-off:

$$V(r) = \min\left(V(r), \frac{N-1}{r}\right), \qquad (16)$$

which is only problematic accuracywise for near neutrals (Cowan, 1981). Note: FAC additionally smooths V(r) around the switch-over r — the same smoothing has been adopted by AS.

Aside: slight differences in the FAC DHS potential arise because the Latter cut-off in equation (16) was originally (e.g. to at least 2020) applied to the direct potential only. Subsequently (e.g. by 2023) it was applied to the total (direct-plus-exchange), Gu (2023).

This is likely of no consequence to the general user, given that AS ultimately uses kappa-averaged orbitals, but it is possible for the expert user to recover this earlier operation if necessary. Note, the look-up table and smoothing are unchanged between '2020' and '2023' operation.

 $^{^4{\}rm TBD}:$ allow AS to optimize FAC_AHX itself using the usual AS optimization flags save that there is a single scaling and variational parameter (so NLAM/NVAR are ignored.)

Above, we plot above various effective charges

$$Z(r) = Z_0(r) - rV(r), (17)$$

where $Z_0(r)$ is the finite nucleus charge, for Be-like Ar¹⁴⁺ to illustrate.

We see that the AS-FAC2023 and FAC2023 curves are indistinguishable.

This completes the detailing of the **current** unique FAC potential described by Gu (2023) now available in v30.x of AS. It can be switched-on by adding FAC='YES'⁵ to SMINIM again together with the NOCC valence occupation numbers such that $\sum_{\alpha} \omega_{\alpha} = N$ still, so as to obtain screening by N-1 electrons. Again, **AS** itself sets all flags necessary to align its operation with that of FAC.

The default AS implementation follows the FAC v1.1.5 2023.03.23 Dirac-Hartree-Slater implementation.

4 Detailed Benchmarking of AS-FAC against FAC

We consider a model problem, involving only s-orbitals, then we should be solving the same Dirac equation and thus obtain the same epsilon orbital energies. We compare AS results obtained by 1/ calculating the FAC potential within AS, 2/ reading the FAC potential written by FAC GetPotential, 3/ running FAC itself. The same physical occupation nos are used in AS and FAC (via AvgConfig & OptimizeRadial). All potentials use the FAC2023 Dirac-Hartree-Slater local exchange form.

AS-FAC vs FAC epsilons (Ryd) for H-like, He-like & Be-like Ar, ground configs only.

| H-like: | AS-FAC | FAC2023 | Diff |
|---------|--------------|-------------|---------|
| 1s: | -325.408963 | -325.408872 | 0.00011 |
| | ASreadFACpot | | |
| • | -325.408954 | | |

There is nothing to iterate here since there is just the nuclear potential. We both appear to use a similar Fermi finite nucleus. The difference between AS-FAC and FAC epsilons is likely down to numerics. (The AS radial mesh accurate to a few e-6 Ryd here.)

| He-like: | AS-FAC | | FAC2023 | Diff |
|----------|--------------|---|-----------------|---------|
| is^2: | -302.23469 | | -302.235963 | 0.00127 |
| - | ASreadFACpot | : | recommended 8 s | s.f. |
| • | -302.23604 | | | |
| • | ASreadFACpot | : | FAC2023 default | 6 s.f. |
| · | -302.22828 | | | |

The AS-FAC epsilons here are converged to 1.d-5 Ryd. AS-FAC is not sensitive to the starting potential. (The AS radial mesh here is accurate to a few e-5 Ryd.)

| Be-like: is^2 2s^2: | -283.58005 | | FAC2023 -283.580272 -62.139234 recommended 8 s. | Diff 0.00022 0.00294 |
|------------------------|--------------|---|--|----------------------------|
| | ASreadFACpot | : | recommended 8 s. | f. |
| • | -283.58031 | | | |
| • | -62.13925 | | | |
| • | ASreadFACpot | : | FAC2023 default | 6 s.f. |
| • | -283.57717 | | | |
| • | -62.13889 | | | |

When we read the FAC potential produced by GetPotential into AS then the AS epsilons are in very close agreement with the original FAC ones. (This is provided we add more significant figures to the GetPotential output file, FAC 2020.04.20 only uses 4 s.f. while FAC 2023.03.23 uses 6 s.f., but 8 s.f. is recommended for AS to achieve closest agreement with the native FAC epsilons.)

⁵FAC='NEW' is also supported to contrast with 'OLD'.

Differences in epsilon (e.g. for non s-orbitals) should not translate into as large differences in level energies of course since we are just playing with perturbation theory. It appears that epsilons are not the main source of differences in level energies.

For example, He-like $1s^2 + 1s2s$ (with 1s and 2s occ. nos 1.5 and 0.5) gives

| Epsilo İs: 2s: | ons | | | AS-FAC -306.08865 -74.26285 | FAC2023 -306.09173 -74.26311 | Diff 0.0031 0.0003 |
|-------------------------------------|--------------------------|----------------------------|----------------------------|--|---|------------------------------------|
| Level 2*S+1 . 1 . 3 . 1 | ener L O O O | rgies 2J 0 2 0 | (inc. CF 1 2 2 | QED) AS-FAC -628.0816091 228.0786505 229.6389177 | FAC2023 -628.0882704 228.0653256 229.6260898 | Diff 0.0067 0.0133 0.0128 |
| Level 2*S+1 . 1 . 3 . 1 | ener L 0 0 0 | rgies 2J 0 2 0 | (exc. CF 1 2 2 | QED) AS-FAC -628.2393362 228.1476417 229.7079070 | FAC2023 -628.2476403 228.1319116 229.6927067 | Diff 0.0083 0.0157 0.0152 |

So, apparently down to the non-relativistic electron-electron integrals..?

5 A Case Study: O^{5+} KLn autoionization rates

We compare various AS and FAC results for 3 autoionizing levels which we (Foster et al 2023) have found to be sensitive to the calculational method and which contribute strongly to the He-like satellite lines.

The three levels in question are:

1: $1s2p(^{1}P)3d^{2}F_{7/2}$

2:
$$1s2p(^{1}P)4d^{2}F_{7/2}$$

3: $1s2p(^{1}P)4p^{2}D_{5/2}$

Our original calculations with AS made use of the fully-relaxed orbital basis and the historic STO model potential and are denoted AS-RLX.

We examined the use of several sets of occupation numbers for the unique FAC potential. We label them by XX = 3, 2A and 2B, where 3 represents the Li-like stage and 2X the He-like, as follows:

1s 2s 2p 3 : 2 1/3 2/3 2A: 1 1/3 2/3 2B: 2 0 0

Note, FAC then splits the 2/3 2p occupation number as 1/3 and 1/3 for the $2p_{1/2}$ and $2p_{3/2}$ orbitals.

The AS input file can be found in the Appendix.

We label the corresponding results AS-FACXX and FACXX, where AS-FACXX denotes an AS calculation using the unique DHS FAC potential detailed in Section 3 and FACXX denotes a FAC calculation (Zhang 2022).

 O^{5+} KLn autoionization rates in units of $10^{13}/s$.

| AS-F | RLXO AS | S-RLX1 | AS-SCCA | | -Hartree-S AS-FAC2A | Slater AS-FAC2B | FAC3 | 2023 FAC2A | FAC2B |
|------|---------|--------|-------------------------|---------------------------|------------------------|-------------------------|-------|-------------------------|-------|
| 2: C | 0.378 | 0.473 | 0.785 0.406 0.880 | $0.510 \\ 0.262 \\ 0.526$ | 0.459 | 0.775 0.428 1.026 | 0.255 | 0.833 0.460 1.108 | 0.431 |

Notes:

AS-RLX denotes fully-relaxed AS results obtained using non-unique potentials and orbitals, but no iteration, just form the STO potentials and generate a set of orbitals.

0 denotes just a Hartree STO potential.

1 denotes Hartree plus local exchange potential.

AS-SCCA is AS-RLX but iterated to self consistency.

While the AS-SCCA results would appear to contain the most physics, AS-RLX0 is a robust result which can use optional scaling scaling parameters. Introducing exchange (AS-RLX1) without self-consistency is poorer.

Regarding the AS-FAC and FAC results, using occupation numbers for the recombining ion gives much better agreement with the relaxed results. However, those for the recombined ion are often preferred for dielectronic recombination because of its sensitivity to the radiative rates — see the FAC Manual 1.1.5, Section 4.2, Q5 (Gu 2016).

Further comparisons between AS-FAC and FAC, for the effect of *n*-mixing on the dielectronic and trielectronic recombination of Ar^{14+} , and the application of Relativistic Many-Body Perturbation Theory (RMBPT) to both, can be found in the paper by Zhang *et al* (2023).

6 Recapitulation

We can now provide answers to the questions raised at the start of this document:

Q. 'Why are you not using FAC?'

A. 'AS is potentially more flexible than FAC.'

and

Q. 'Why do your results differ from those of FAC?'

A. 'If we use the same potential as FAC then our AS results are in close agreement with FAC's, but we argue for the use of a more flexible, physical, set of potentials and orbitals.'

7 Acknowledgements

Thanks go to Ming Feng Gu for many conversations regarding the exact form of the present FAC potential and to Chunyu Zhang for clarifying certain aspects of FAC operation and providing the FAC results shown here.

Further comparisons between AS-FAC and FAC, for the effect of *n*-mixing on the dielectronic and trielectronic recombination of Ar^{14+} , and the application of Relativistic Many-Body Perturbation Theory (RMBPT) to both, see Zhang *et al* (2023).

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9 Appendix

Here we detail and comment-on the AS input files.⁶ The file below is for AS-FAC3. AS-FAC2A and AS-FAC2B can be obtained by moving the required occupation numbers to the top of the list.

The AS-RLX input can be obtained by setting BASIS='RLX' and adding &END after INUKE=1. A one-off (per NZION) structure run is necessary (set MXCCF=0) to generate the TERMS/LEVELS file necessary for AS to set-up the continuum interpolation energy mesh.⁷

A.S. He-like 1-2,n: n=2-nxtra RUN=' ' RAD='YES' CUP='ICR' MXVORB=3 MXCONF=3 MXCCF=2 BASIS='' &SALGEB ICFG=10 NXTRA=5 LXTRA=4 lcon=6 KUTSO=0 KUTOO=1 KUTSS=999 & END 102021 2 0 0 !These are just the usual MXCONF=3 He-like N-electron target configs $\begin{smallmatrix}1&1&0\\1&0&1\end{smallmatrix}$ 1 0 0 !minimum occupation (N+1-electron) 222 !maximum occupation (N+1-electron) !We have two N+1 base configs (MXCCF=2) plus no. of promotions. 2 1 0 1 !The choice is not unique. One can get the same set of configs from a 1 1 1 1 !different set of occupation constraints, base configs, and promotions. &SMINIM NZION=-8 PRINT='FORM' QED=1 INUKE=1 NOCC=3 PPOT='SCCA' FAC='YES' & END 2.0 0.3333 0.6667 !FAC3 1.0 0.3333 0.6667 !FAC2A 2.0 0.0 !FAC2B 0.0 &SRADCON MENG=-15 & END !Requires a TERMS/LEVELS structure file (case LS/IC) -0.1 -1.0

Notes:

0/ Everything is free-formatted (and case insensitive.)

1/ We use NXTRA and LXTRA to extend the orbital definition list (AtomDB did not use 5g...) They are discussed in Sec.4a NAMELISTs of the AS WRITEUP file. Update to AS v28.46.9 to avoid having to set the continuum expansion (L=0,..LCON-1) via LCON — prior did not check for LXTRA. Even then AS is too conservative/not clever enough and sets more than we need here because all it looks at are the nl-values, not how they are used in configs. So it sets-up to allow for 1s5g+continuum — better safe than sorry. The overkill not an issue for He-like, but in general...

2/ We use promotion rules (via ICFG=10) to generate the N+1 configs, it is not worth it for the N-electron. We could have flagged use of promotion rules for N-electron as well (ICFG=11), but with zero promotions, to obtain the same effect. *I would set this an as exercise for my students! ICFG/10 is gives the MXCCF N+1-electron ICFG value and MOD(ICFG,10) gives the MXCONF N-electron ICFG. So, here ICFG=0 for the N-electron MXCONF spec. (the usual) and ICFG=1 for N+1-electron MXCCF. ICFG is also discussed in Sec.4a NAMELISTs of the AS WRITEUP file. 3/ 'Promotion' also includes 'demotion'.

4' Two base configs can generate the same new config. Duplicates are dropped, of course!

5/ If NXTRA,LXTRA generate too many orbitals then the Eissner notation still used in oic is replaced by an asterisk. adasdr can probably handle it, adaspe probably not. The max number of non-asterisked is 61. *Question for Students: why 61?

6/ All possible relativistic interactions have been switched-on, except that 2-body fine-structure are restricted to being within a configuration (KUTSS=999) for speed, without loss of accuracy (KUTSS=-999 would include between).

*** It is not necessary to switch-on these relativistic interactions when FAC='YES'. ***

*** AS will automatically switch them on so as to align its operations with FAC's. ***

*** But they *are* required if BASIS='RLX' because FAC='NO' then. ***

7/ Again, QED and INUKE are required if BASIS='RLX' (and PPOT='SCCA', if so desired).

AS Working Note No. 3.

May 30, 2024

 $^{^{6}}_{-}$ The periods '.' at the start of blank lines are there *only* to space-out text under latex verbatim.

⁷The approximation method used does not matter for this set-up.