

Non-thermal recombination – a neglected source of flare hard X-rays and fast electron diagnostics (Corrigendum)

J. C. Brown¹, P. C. V. Mallik¹, and N. R. Badnell²

¹ Department of Physics and Astronomy, University of Glasgow, Glasgow G12 8QQ, UK
 e-mail: pmallik@astro.gla.ac.uk

² Department of Physics, University of Strathclyde, Glasgow G4 0NG, UK

A&A, 481, 507–518 (2008), DOI: [10.1051/0004-6361/20078103](https://doi.org/10.1051/0004-6361/20078103)

ABSTRACT

Brown and Mallik (BM) recently claimed that non-thermal recombination (NTR) can be a dominant source of flare hard X-rays (HXR) from hot coronal and chromospheric sources. However, major discrepancies between the thermal continua predicted by BM and by the Chianti database as well as RHESSI flare data, led us to discover substantial errors in the heuristic expression used by BM to extend the Kramers expressions beyond the hydrogenic case. Here we present the relevant corrected expressions and show the key modified results. We conclude that, in most cases, NTR emission was overestimated by a factor of 1–8 by BM but is typically still large enough (as much as 20–30% of the total emission) to be very important for electron spectral inference and detection of electron spectral features such as low energy cut-offs since the recombination spectra contain sharp edges. For extreme temperature regimes and/or if the Fe abundance were as high as some values claimed, NTR could even be the dominant source of flare HXR, reducing the electron number and energy budget, problems such as in the extreme coronal HXR source cases reported by e.g. Krucker et al.

Key words. atomic processes – Sun: corona – Sun: flares – Sun: X-rays, gamma rays – errata, addenda

1. Summary

Brown & Mallik (2008, 2009) (BM) recently argued that, for hot sources, recombination of non-thermal electrons (NTR) onto highly ionised heavy ions is not negligible compared to non-thermal bremsstrahlung (NTB) as a source of flare hard X-rays (HXR) and so should be included in modelling non-thermal HXR flare emission. They further claimed that, in some cases, NTR can be much larger than NTB with important consequences for flare physics. In view of major discrepancies between BM results for the *thermal* continua and those of the Chianti code (e.g. Dere et al. 2009) and of RHESSI (Lin et al. 2002) solar data, we critically re-examined the BM analysis and discovered substantial errors in the heuristic expression used by BM to extend the Kramers expressions beyond the hydrogenic case. Here we summarise the main resulting modifications to their equations and results (now validated against detailed calculations) and their conclusions concerning the importance of NTR. The BM results are correct for NTR onto Fe 26+ and a factor of 2 too high for Fe 25+ so that, at high enough $T > 40$ MK for these to exist, NTR does strongly dominate NTB in the deka-keV range. However, at such T , thermal continuum dominates NTR and NTB in this energy range unless the non-thermal electron density is a very large fraction, f_c , of the total as in some coronal HXR sources (e.g. Krucker et al. 2008). At $T \approx 10$ –30 MK, the dominant Fe ions are Fe 22+, 23+, 24+ for which BM overestimated NTR emission by around an order of magnitude. Thus NTR in typical hot flare plasmas does *not* dominate over NTB. However, in many cases it can be comparable and, especially as NTR includes sharp edge features, very important in inversions of photon spectra to derive electron spectra. In cosmic

situations of higher Fe abundance, or in flare sources of higher coronal Fe abundance than BM used (cf. Feldman et al. 2004), NTR could be a dominant source of flare HXR in some energy/temperature regimes. Full details are available in Mallik (2010). The BM results were mainly for the widely used, though unphysical, electron flux spectrum $F(E) \propto E^{-\delta}$ at $E \geq E_c$ and zero below that. For generality of our corrections here we also give the NTR expression for general $F(E)$.

2. Amended equations and results

The core error in BM was their use, in the Kramers cross section and edge energies, of quantum number $m = 1$ for the *first* unfilled level instead of its principal quantum number n . For Fe 26+ and 25+, $n = m = 1$, but for Fe 24+ to Fe 17+, for example, $m \neq 1$ but $=2$. In ionic species which already have 2–9 electrons present, the smallest n value n_{\min} is 2, and for 10 or more electrons, it is $n_{\min} = 3$ and so on. (Recombination rates to levels with $n > n_{\min}$ still fall off as $1/n^3$, so are rather small in comparison). The consequences of this BM error are:

- since, for typical hot flare temperatures of 20–30 MK, Fe 24+ and Fe 23+ are the most abundant Fe ions, $n_{\min} = 2$, not 1, and the magnitude of the recombination emission is down compared to BM by a factor of around $1/n_{\min}^3 = 1/8$;
- the recombination edges for these species are no longer at $Z_{\text{eff}}^2\chi$, or $Z_{\text{eff}}^2\chi + E_c$ in the presence of a low energy cut-off E_c , but at $Z_{\text{eff}}^2\chi/4$ or $Z_{\text{eff}}^2\chi/4 + E_c$ respectively;
- additionally, the Kramers formula applies to recombination into an empty shell. For partially filled n -shells, a “vacancy factor”, p_n , has to be applied. If all recombinations to that

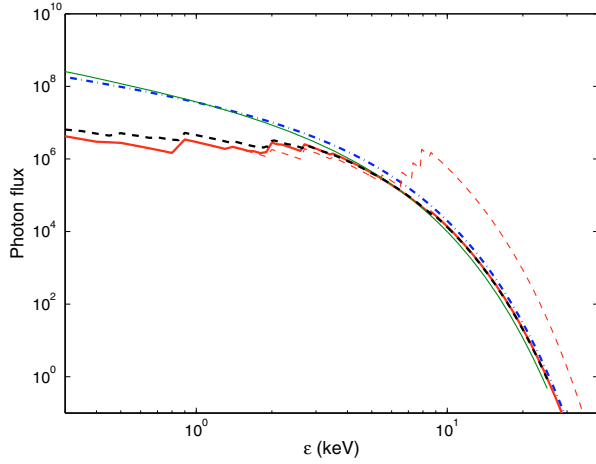


Fig. 1. The revised thermal model spectra (dot-dash blue is TB; thick solid red is TR) compared with Chianti's (thin solid green for TB; dashed black for TR) at $T = 20$ MK. Also included is the erroneous TR spectrum (thin dashed red) from [Brown & Mallik \(2008\)](#). The major discrepancy caused by the Fe edges in BM have been suitably resolved in our revision. The smaller discrepancies are due to differences in the Kramers' and measured cross-sections as well as the possibility of recombination to higher levels than $n = n_{\min}$. Courtesy E. Landi for the Chianti spectra.

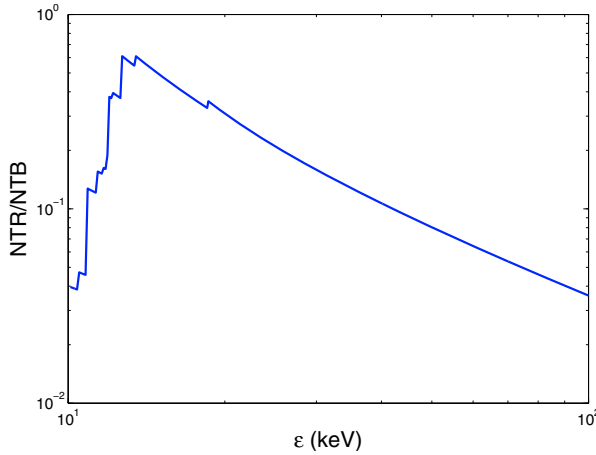


Fig. 2. Thin-target NTR:NTB ratio for $E_c = 10$ keV, $\delta = 5$ and $T = 20$ MK. This plot can be compared to the bottom left panel of [Brown & Mallik \(2008\)](#). Although shapes are similar, the revised ratio values for the prominent Fe edges are about a factor of 8 lower.

level had equal rate, p_n would be N_v/N_n , where $N_n = 2n^2$ is the total electron occupation number of the shell and N_v the number of states unoccupied – e.g. for a He-like ion, $p_1 = 1/2$ while for a partially filled $n = 2$ shell, $N_2 = 8$. However, recombination into $n = 2$ in the Kramers formula is actually dominated by the 6 p -states, at the electron energies of interest here so we can take $N_2 = 6$ and N_v the number of unoccupied $2p$ states, i.e. $p_2 = 1$ for Li and Be-like initial ions and $p_2 = 5/6, 4/6, \dots, 1/6$ for B- through F-like initial ions. Comparisons of such modified Kramers cross sections have been made with the results of detailed calculations using the AUTOSTRUCTURE code (cf. [Badnell 2006](#)) for initial H-like through to F-like Fe ions and agreement to within 20% was obtained for our new results using this estimate.

Revisions of the recombination cross-section expressions given in Eqs. (11) and (12) of [Brown & Mallik \(2008\)](#) have to be carried through to all other recombination expressions, e.g.

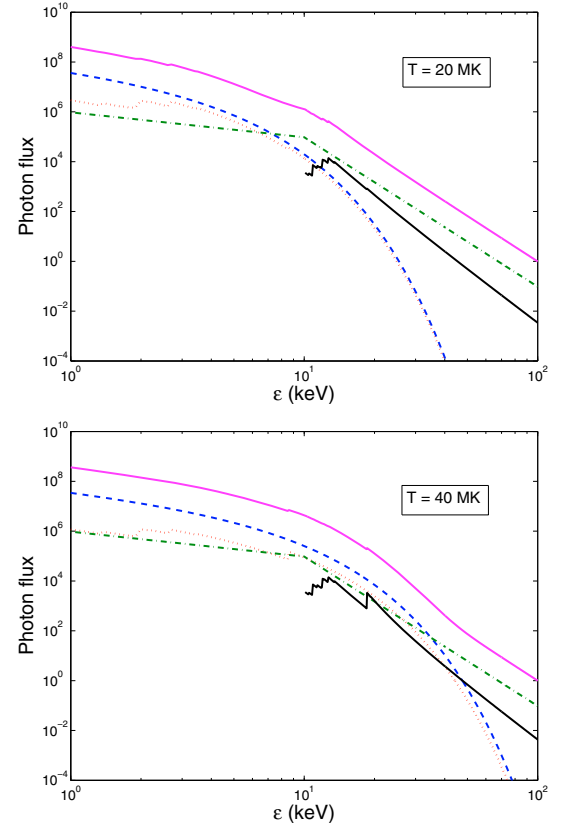


Fig. 3. The revised thin-target spectra for 2 different temperatures with $f_c = 0.1$. The dashed blue curve is TB, dotted red is TR, dot-dash green is NTB and solid black is NTR. The solid magenta curve is the total flux multiplied by 10. Note that the non-thermal emission measure, $EM_c = f_c EM$, where $EM = 2ALn_p^2$ is the thermal emission measure.

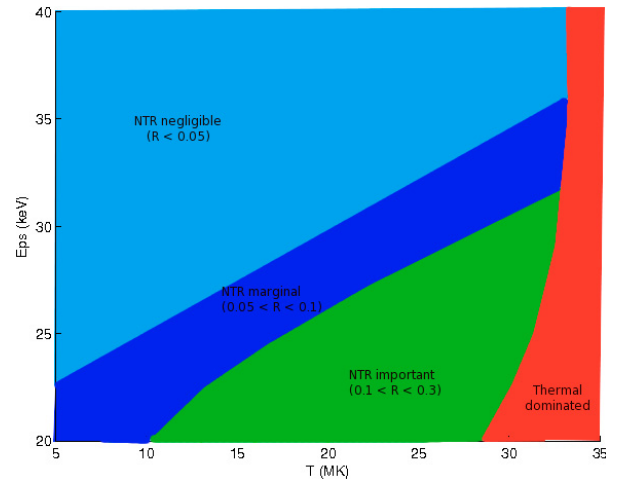


Fig. 4. Regime plot with our revised model showing the relevant areas of importance in the (ϵ, T) domain for $E_c = 10$ keV, $\delta = 5$ and $f_c = 0.1$. Note the large domain where NTR contributes 0.1–0.3 of the continuum. For bigger f_c the NTR importance domain increases further.

Eqs. ((16), (19), (B.2), (B.5), (B.12)) in [Brown & Mallik \(2008\)](#) and Eq. (2) in [Brown & Mallik \(2009\)](#), to show the sum over $n \geq n_{\min}$, with $n_{\min} \neq 1$ in general. So, the revised thin-target non-thermal recombination expression (replacing Eq. (B.2) of

Table 1. Significance of Fe 25+ and 26+ at 4 different temperatures for $\epsilon = 20$ keV and $\delta = 5$.

| T (MK) | A_Z (Fe 25+) | A_Z (Fe 26+) | ζ_{RZ} (Fe 25+) | ζ_{RZ} (Fe 26+) | R (Fe 25+) | R (Fe 26+) |
|-------------|----------------------|-----------------------|---------------------------------|---------------------------------|----------------------|----------------------|
| 20 | 2.4×10^{-7} | 2.5×10^{-10} | 4.7×10^{-2} | 1.1×10^{-4} | 1.0×10^{-2} | 3.4×10^{-5} |
| 30 | 3.5×10^{-6} | 4.5×10^{-8} | 0.68 | 2.1×10^{-2} | 0.15 | 6.5×10^{-3} |
| 40 | 6.7×10^{-6} | 2.8×10^{-7} | 1.3 | 0.13 | 0.28 | 4.0×10^{-2} |
| 50 | 1.7×10^{-5} | 1.3×10^{-6} | 3.3 | 0.59 | 0.70 | 0.18 |

Brown & Mallik 2008) is (all notation as in BM):

$$J_{\text{Rthin}}(\epsilon) = (\delta - 1) \frac{32\pi}{\sqrt{3}\alpha} \frac{r_e^2 \chi^2}{\epsilon} \frac{2n_p A L F_c}{E_c^2} \sum_{Z_{\text{eff}}} \sum_{n \geq n_{\text{min}}} p_n \zeta_{\text{RZ-eff}} \frac{1}{n^3} \times \left[\frac{\epsilon - V_{Z_{\text{eff}}}/n^2}{E_c} \right]^{-\delta-1} ; \epsilon \geq E_c + V_{Z_{\text{eff}}}/n^2 \times 0 ; \epsilon < E_c + V_{Z_{\text{eff}}}/n^2 \quad (1)$$

for the cut-off power law $F(E)$ and with $V_{Z_{\text{eff}}} = Z_{\text{eff}}^2 \chi$. We also give the corrected thin target expression for general $F(E)$ in Eq. (4) to enable readers to extend our results.

The revised thermal recombination expression (from Eq. (B.5) of Brown & Mallik 2008) as a function of photon energy ϵ is:

$$J_{\text{Rtherm}}(\epsilon) = \sqrt{\frac{2\pi}{27m_e}} \frac{64r_e^2 \chi^2}{\alpha} \frac{2n_p^2 A L}{\epsilon (kT)^{3/2}} \sum_{Z_{\text{eff}}} \sum_{n \geq n_{\text{min}}} p_n \zeta_{\text{RZ-eff}} \frac{1}{n^3} \times \exp\left(\frac{V_{Z_{\text{eff}}}/n^2 - \epsilon}{kT}\right) ; \epsilon \geq V_{Z_{\text{eff}}}/n^2 \times 0 ; \epsilon < V_{Z_{\text{eff}}}/n^2. \quad (2)$$

The thick-target NTR expression (revised from Eq. (B.12) of Brown & Mallik 2008) is (power-law case):

$$J_{\text{Rthick}}(\epsilon) = \frac{32\pi r_e^2 \chi^2 \mathcal{F}_{\text{oc}}}{3\sqrt{3}\alpha} \frac{1}{K\epsilon} \sum_{Z_{\text{eff}}} \sum_{n \geq n_{\text{min}}} p_n \zeta_{\text{RZ-eff}} \frac{1}{n^3} \times \left[\frac{\epsilon - V_{Z_{\text{eff}}}/n^2}{E_{\text{oc}}} \right]^{-\delta_o+1} ; \epsilon \geq E_{\text{oc}} + V_{Z_{\text{eff}}}/n^2 \times \left[\frac{E_{\text{oc}} - V_{Z_{\text{eff}}}/n^2}{E_{\text{oc}}} \right]^{-\delta_o+1} ; V_{Z_{\text{eff}}}/n^2 < \epsilon < E_{\text{oc}} + V_{Z_{\text{eff}}}/n^2 \times 0 ; \epsilon \leq V_{Z_{\text{eff}}}/n^2. \quad (3)$$

(Thick target results are for the total emission rates over continuously injected electron collisional lifetimes). Note that Eqs. (1) and (3) also correct the following typos in Eqs. (B.2) and (B.12) of Brown & Mallik (2008):

- the term $\zeta_{\text{RZ-eff}}$ was accidentally put outside the \sum in Eq. (B.2) of BM.
- it should be χ^2 in the numerator for Eq. (B.2) of BM and not just χ .

- the terms $m_e c^2$ in the numerator and E_{oc} in the denominator of Eq. (B.12) of BM do not actually exist and were erroneously carried over from some other formulae.

For an arbitrary electron spectrum $F(E)$ in source volume V , replacing Eq. (11) in BM 2008 and scaled up by the source volume,

$$J_{\text{thin}}^{F(E)}(\epsilon) = \frac{32\pi r_e^2 \chi^2 n_p V}{3\sqrt{3}\alpha \epsilon} \sum_Z \sum_{n \geq n_{\text{min}}} p_n \frac{A_Z Z^4}{n^3} \frac{F(\epsilon - Z^2 \chi/n^2)}{\epsilon - Z^2 \chi/n^2}. \quad (4)$$

The ionisation equilibrium used was a fit to the standard steady state coronal collisional ionisation, optical depths being negligible. In Table 1, we have listed the abundances and ζ_{RZ} for Fe 25+ and Fe 26+ (Arnaud & Raymond 1992) for a set of four temperatures, and these can be seen in addition to the relevant values in Tables 1 and 2 of Brown & Mallik (2008), where these values are listed for fully ionised Fe (Fe 26+) at $T \gg 100$ MK and for the most abundant Fe species at 20 MK. Here we have also included the ratio of non-thermal free-bound (due to only Fe 25+ and 26+ respectively) to total free-free flux, $R = J_{\text{NTRFe26+,25+}}/J_{\text{NTB}}$ (from Eq. (2) of Brown & Mallik 2009), at $\epsilon = 20$ keV and $\delta = 5$ to illustrate at what temperatures Fe 25+ and Fe 26+ start becoming significant contributors to the NTR flux.

Acknowledgements. We gratefully acknowledge the financial support of a UK STFC Rolling Grant (JCB), of a Dorothy Hodgkin's Scholarship (PCVM), an ISSI Grant (JCB), and an STFC Grant (NRB). Discussions with E.P. Kontar, R.A. Schwartz, and E. Landi were invaluable in drawing attention to problems with the BM results. We are grateful to the referee in helping us condense and improve our report of the amendments.

References

- Arnaud, M., & Raymond, J. 1992, ApJ, 398, 394
 Badnell, N. R. 2006, ApJS, 167, 334
 Brown, J. C., & Mallik, P. C. V. 2008, A&A, 481, 507
 Brown, J. C., & Mallik, P. C. V. 2009, ApJ, 697, L6
 Dere, K. P., Landi, E., Young, P. R., et al. 2009, A&A, 498, 915
 Feldman, U., Dammasch, I., Landi, E., & Doschek, G. A. 2004, ApJ, 609, 439
 Krucker, S., Wuelser, J.-P., Vourlidas, A., et al. 2008, 12th European Solar Physics Meeting, Freiburg, Germany, held September, 8–12, <http://espm.kis.uni-freiburg.de/>, 2.84, 12, 2
 Lin, R. P., Dennis, B. R., Hurford, G. J., et al. 2002, Sol. Phys., 210, 3
 Mallik, P. C. V. 2010, Ph.D. Thesis, University of Glasgow