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Dielectronic recombination between hyperfine levels of the ground state of Bi^{82+}

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Dielectronic recombination cross sections between the hyperfine levels of the ground state of Bi^{82+} are calculated in a hydrogenic approximation. The radiative decay rates from the doubly excited resonances $1s(F=5)nl$ are found to be orders of magnitude stronger than the autoionizing decay rates to the $1s(F=4)$ continuum. The associated dielectronic recombination cross sections are strongly peaked towards zero energy, but their overall magnitudes are extremely small compared to the radiative recombination background.

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New opportunities for the study of hyperfine interactions in highly charged atomic ions have been made possible by the construction of heavy-ion storage-cooler rings [1]. For circulating cooled beams of hydrogenlike atoms, accurate measurements of the $1s$ hyperfine splitting may be obtained after first populating the upper $1s$ hyperfine level by either resonant collinear laser excitation or electron-spin-exchange collisions in the cooler device. The principal loss mechanism for the hydrogenlike atomic beam is expected to be both radiative and dielectronic recombination. Of course, a measurement of the total recombination cross section for the highly charged ions is of interest in its own right. In this paper we attempt to estimate the size of the dielectronic recombination cross section for Bi^{82+} by resorting to high-Z hydrogenic approximations. Although estimates are only of a qualitative nature, we do find that the natural linewidth broadened resonance peak heights coming from dielectronic recombination are 2 orders of magnitude smaller than the smooth background due to radiative recombination.

An estimate of the strength of the dielectronic recombination cross section between hyperfine levels of the ground state of Bi^{82+} is made in the following paragraphs. The energy separation between the ground-state hyperfine levels for hydrogenic atoms is given approximately by [2]

$$\Delta E=\tfrac{4}{3}Z^3\frac{g(I+1/2)}{c^2(1836.1)}\,, \tag{1}$$

where Z is the atomic number and c is the speed of light (in atomic units). For the $A=209$ isotope of Bismuth, the nuclear spin $I=9/2$ and nuclear moment ratio $g=0.91$, resulting in an energy separation of 2.7 eV. For high-Z ions the hyperfine transitions may thus be induced by powerful optical lasers.

For dielectronic recombination we considered the following reaction pathways:

$$e^-+\mathrm{Bi}^{82+}(1sIF')\rightarrow \mathrm{Bi}^{81+}(1sIFnljJ) \tag{2}$$

and

$$\mathrm{Bi}^{81+}(1sIFnljJ)\rightarrow \mathrm{Bi}^{81+}(1sIFn'l'j'J')+h\nu \rightarrow \mathrm{Bi}^{82+}(1sIF')+e^-\,, \tag{3}$$

where $F'=4$ and $F=5$. In the isolated resonance approximation, the dielectronic capture cross section is given by

$$\sigma_{\rm cap}(1sIF' \rightarrow 1sIFnljJ) = \frac{\pi^2}{2E}\frac{2J+1}{2F'+1} \times A_a(1sIFnljJ \rightarrow 1sIF')L(E)\,, \quad (4)$$

where the Lorentz profile is given by

$$L(E) = \frac{(1/2\pi)\Gamma}{(E-E_n-\Delta E)^2+\frac{1}{4}\Gamma^2}\,, \quad (5)$$

$E_n = -(Z-1)^2/2n^2$, E is the incident electron energy, and the total decay width is given by

$$\Gamma = A_a(1sIFnljJ \rightarrow 1sIF') + \sum_{n',l',j',J'} A_r(1sIFnljJ \rightarrow 1sIFn'l'j'J')\,. \quad (6)$$

The total dielectronic recombination cross section is thus written

$$\sigma_{\rm DR} = \sum_{n,l,j,J} \sigma_{\rm cap}(1sIF' \rightarrow 1sIFnljJ)B_{\rm RS}\,, \quad (7)$$

where the branching ratio for radiative stabilization is given by

$$B_{\rm RS} = \sum_{n',l',j',J'} A_r(1sIFnljJ \rightarrow 1sIFn'l'j'J')/\Gamma\,. \quad (8)$$

Using the hyperfine energy separation from Eq. (1) and the hydrogenic energy expression, E_n, for the doubly excited resonances, we find that the minimum n value for recombination is 183. A more realistic determination of ΔE will change the minimum n value from 183, but will not change the qualitative nature of the final results. Thus $n' < 183$ in the sum found in Eq. (8). We also restrict the sum in Eq. (6) to $n' < 183$, thus ignoring any contributions from cascade (which are small since the sum is dominated by the lowest few n').

First-order perturbation theory involving only the electrostatic interaction between electrons is used to evaluate the autoionization rates, A_a, found in Eqs. (4) and (6). Making use of the Racah algebra of tensor operators [3], the direct term in the multipolar expansion for the electrostatic matrix element is found to vanish, while the exchange term survives. What is involved is the recoupling of the spin of the 1s electron to the nuclear moment, and this requires the exchange term to flip the spin of the 1s electron. The autoionization rate of a doubly excited resonance may thus be written as

$$A_a(1sIFnljJ \rightarrow 1sIF') = 4\sum_{j'} \frac{(2F+1)(2F'+1)(2j+1)(2j'+1)}{(2l+1)^2} \begin{Bmatrix} I & 1/2 & F \\ 1/2 & l & j \\ F' & j' & J \end{Bmatrix}^2 R_l^2\,, \quad (9)$$

where the curly brackets indicate a standard 9-j symbol. The radial exchange integral is given by

$$R_l = \int_0^\infty dr_1 \int_0^\infty dr_2 P_{1s}(r_1)P_{nl}(r_1) \times P_{1s}(r_2)P_{El}(r_2)\frac{r_<^l}{r_>^{l+1}}\,, \quad (10)$$

where $r_{\gtrless} = {}^{\rm min}_{\rm max}\{r_1, r_2\}$ and $P_{nl}(r)$ are radial wave functions. The normalization of the continuum wave function, $P_{El}(r)$, is chosen as $(2E)^{-1/4}$ times a sine function. We note that the integral over r_1 in Eq. (10) involving the 1s orbital and a high Rydberg nl orbital is small and will in-

TABLE I. Autoionization and radiative rates for Bi^{82+}.

(a) $A_a(1sIFnljJ \rightarrow 1sIF')$
$I=9/2$, $F'=4$, $F=5$

n	l	j	J	Rate ($\sec^{-1}$)
183	0	1/2	9/2	8.90×10^8
	1	1/2	9/2	1.43×10^8
			11/2	3.17×10^8
		3/2	7/2	5.23×10^8
			9/2	3.80×10^8
			11/2	2.06×10^8
	2	3/2	7/2	5.31×10^5
			9/2	1.21×10^6
			11/2	2.05×10^6
			13/2	3.03×10^6
		5/2	5/2	4.17×10^6
			7/2	3.64×10^6
			9/2	2.96×10^6
			11/2	2.12×10^6
			13/2	1.14×10^6

(b) $\sum_{n',l',j',J'} A_r(1sIFnljJ \rightarrow 1sIFn'l'j'J')$
$I=9/2$, $F=5$

n	l	Rate ($\sec^{-1}$)
183	0	2.90×10^{13}
	1	1.74×10^{14}
	2	9.49×10^{13}

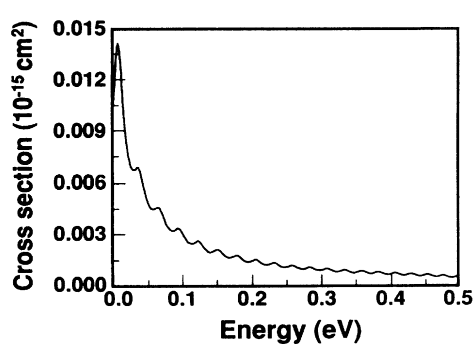

FIG. 1. Dielectronic recombination cross section for Bi^{82+}. The lowest peak is due to resonance contributions involving the $n=183$ shell.

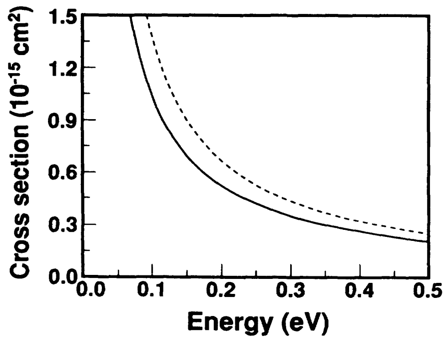

FIG. 2. Radiative recombination cross section for Bi^{82+}. Solid curve: distorted-wave calculation; dashed curve: Bethe-Salpeter formula.

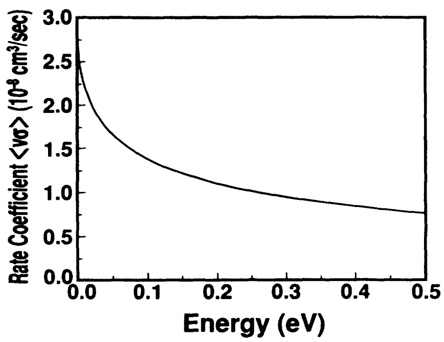

FIG. 3. Total recombination rate coefficient $\langle v\sigma\rangle$ for Bi^{82+}.

variably lead to relatively small values for the resulting autoionization rates.

First-order perturbation theory involving only the dipole interaction is used to evaluate the radiative rates, A_r, found in Eqs. (6) and (8). The radiative rate for a doubly excited resonance to all bound states may be expressed as

$$\sum_{n',l',j',J'} A_r(1sIFnljJ \rightarrow 1sIFn'l'j'J') = \frac{4\omega^3}{3c^3}\frac{l_>}{2l+1}D^2_{l\rightarrow l'} , \tag{11}$$

where ω is the frequency of the light emitted, $l_> = \max\{l,l'\}$, and $l'=l\pm 1$. The radial dipole integral is given by

$$D_{l\rightarrow l'} = \int_0^\infty P_{nl}(r)P_{n'l'}(r)rdr . \tag{12}$$

The radial exchange and dipole integrals of Eqs. (10) and (12) were evaluated using bound hydrogenic and Coulomb wave functions. Numerical methods developed previously [4] were employed to handle the high Rydberg states involved. Autoionization and radiative rates for Bi^{82+} obtained using Eqs. (9) and (11) are given in Table I for the $n=183$ shell. Even for $l=0$ the autoionization rate is over 4 orders of magnitude smaller than the radiative rate. Due to the exchange nature of the radial Coulomb integrals, the autoionization rates drop sharply with l, i.e., by $l=3$ the largest rate is 4.71×10^3 $\sec^{-1}$. For all practical purposes, $B_{RS}=1.0$ in Eq. (8) and $\sigma_{DR} \propto \sigma_{cap} \propto A_a$. Both the autoionization and radiative rates exhibit pure $1/n^3$ scaling.

The dielectronic recombination cross section between hyperfine levels of the ground state of Bi^{82+} is presented as a function of electron energy in Fig. 1. The $n=183$ shell is the largest peak near zero energy. Contributions from $n=184$ to 202 are also visible stretching off to higher energies. Even when the calculation is extended to include $n\leq 3000$ there is no accumulation peak found at the Rydberg series limit of 2.7 eV. The widths of the resonances shown in Fig. 1 are the natural linewidths due largely to radiative decay. An experimental electron energy resolution of 0.1 eV will wash out all of the structure.

The radiative recombination cross section for the ground state of Bi^{82+} is presented as a function of electron energy in Fig. 2. The solid curve is a distorted-wave calculation obtained using a semirelativistic version of the AUTOSTRUCTURE code [5], while the dashed curve is an estimate obtained using the expression [2]

$$\sigma_{RR} = \frac{1.96\pi^2}{c^3 n^3}\frac{E_1^2}{E(E-E_n)} , \tag{13}$$

where $E_1 = -(Z-1)^2/2$. On comparing Figs. 1 and 2 we find that the radiative recombination cross section is over 2 orders of magnitude larger than the dielectronic recombination cross section in this energy region. The DR peaks are within the width of the line drawn to represent the RR cross section in Fig. 2. To guide experimental efforts we convolute the total recombination cross section for Bi^{82+} with a typical electron-cooler velocity distribution [6] and present the resulting rate coefficient $\langle\sigma v\rangle$ as a function of electron energy in Fig. 3.

In conclusion we find that the dielectronic recombination cross section between hyperfine levels of the ground state of Bi^{82+} is extremely small when compared to the radiative recombination background. We feel that an isolated resonance hydrogenic calculation for the dielectronic recombination cross section is probably accurate to within an order of magnitude. However, there are many remaining facets of the problem which make an accurate calculation much more involved. Among these are the problems of treating the overlapping resonances correctly [7] and including relativistic effects on the resonance energy positions and autoionization rates [8].

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